Calculus 3 – First Order Partial Derivatives

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Recently, I encountered a matrix called jacobian matrix. Jacobian matrix is a matrix that consists of first order partial derivatives of vector value function. Pseudo Inverse of jacobian matrix can be used to solve inverse kinematic problem in robotic field. So what is partial derivatives ? Partial derivative symbolized as ∂ (partial dee). Partial Derivative is a part of calculus. Based on literature :

“a derivative of a function of two or more variables with respect to one variable, the other(s) being treated as constant.”

example function:

$f(x,y) = y^3x + 4x + 5y$

$\partial f/\partial x$ means partial derivative of f(x,y) in respect to x. where we treat y as constant.

$\partial f/\partial x = y^3 + 4$

steps:

- partial derivative ($\partial f/\partial x$) of $y^3x = y^3x / x = y^3$
- partial derivative ($\partial f/\partial x$) of $4x = 4x / x = 4$
- partial derivative ($\partial f/\partial x$) of constant $5y = 0$
- so $\partial \mathbf{f}/\partial \mathbf{x} = \mathbf{y^3 + 4}$

$\partial f/\partial y$ means partial derivative of f(x,y) in respect to y. where we treat x as constant.

$f(x,y) = y^3x + 4x + 5y$

$\partial f/\partial y = 2y^2x + 5$

steps :

- partial derivative ($\partial f/\partial y$) of $y^3x = 2y^2x$
- partial derivative ($\partial f/\partial y$) of contsant $4x = 0$
- partial derivative ($\partial f/\partial y$) of $5y = 5y / y = 5$
- so $\partial \mathbf{f}/\partial \mathbf{y} = \mathbf{2y^2x + 5}$

Another example, partial derivative of this explicit function :

$y = 3x^2 – 5z^2 + 2x^2z – 4xz^2 – 9$

$\partial y/\partial x = ?$

steps :

- $\partial y/\partial x$ of $3x^2 = \mathbf{2.3x^{2-1}} = 6x$
- $\partial y/\partial x$ of $5z^2 = 0$

- $\partial y/\partial x$ of $2x^2z = 2.2x^{2-1}z = 4xz$
- $\partial y/\partial x$ of $4xz^2 = 4z^2$
- $\partial y/\partial x$ of $9 = 0$
- So $\partial y/\partial x$ of $y = 6x + 4xz - 4z^2$

what about $\partial y/\partial z$?

steps:

- $\partial y/\partial z$ of $3x^2 = 0$
- $\partial y/\partial z$ of $5z^2 = 2.5z^{2-1} = 10z$
- $\partial y/\partial z$ of $2x^2z = 2x^2z * 1/z = 2x^2$
- $\partial y/\partial z$ of $4xz^2 = 2.\ 4xz^{2-1} = 8xz$
- $\partial y/\partial z$ of $9 = 0$
- So $\partial y/\partial z$ of $y = -10z + 2x^2 – 8xz$

First Order Partial Derivative of a function that consists of 3 variables ?

Example : $f(x,y,z) = z^4 - x^6y^2 + 5$

$f'_x = ?$

steps :

- $\partial f/\partial x$ of $z^4 = 0$
- $\partial f/\partial x$ of $x^6y^2 = 6x^{6-1}y^2 = 6x^5y^2$
- $\partial f/\partial x$ of $5 = 0$
- thus $\partial f/\partial x$ of $f(x,y,z) = \mathbf{-6x^5y^2}$

$f'_y = ?$

- $\partial f/\partial y$ of $z^4 = 0$
- $\partial f/\partial y$ of $x^6y^2 = x^6\ 2y^{2-1} = x^6\ 2y$
- $\partial f/\partial y$ of $5 = 0$
- thus $\partial f/\partial y$ of $f(x,y,z) = \mathbf{-x^6\ 2y}$

$f'_z = ?$

- $\partial f/\partial z$ of $z^4 = 4z^{4-1} = 4z^3$
- $\partial f/\partial z$ of $x^6y^2 = 0$
- $\partial f/\partial z$ of $5 = 0$
- thus $\partial f/\partial z$ of $f(x,y,z) = \mathbf{4z^3}$